In this latter picture a ray corresponds to a ray ending up exactly n lat positions removed from its origin position

① pick a direction vector
② calculate reflection off 3 sets of orthogonal planes
③ calculate travel distance between successive hits on a single set of planes
④ assign a loss/cm number

this looks amenable to an analytic treatment
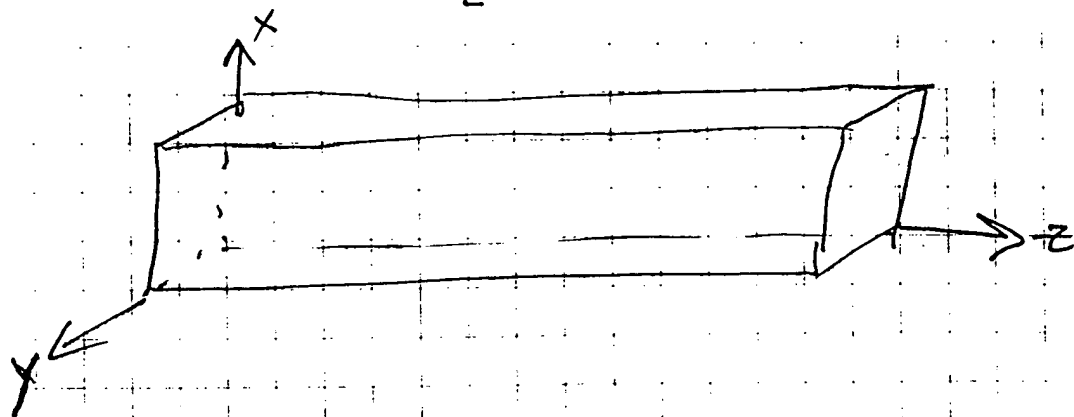
use direction cosines to parameterize ray direction
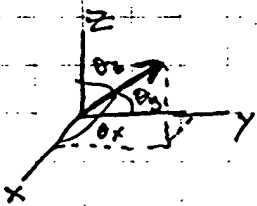
$$(\cos\theta_x, \cos\theta_y, \cos\theta_z) = \frac{(RND1, RND2, RND3)}{\sqrt{RND1^2 + RND2^2 + RND3^2}}$$

let Δx, Δy, and Δz denote slab dimension or plane spacing.

6-Sep-9?

From this point of view it doesn't matter what position a ray is launched from, only its direction, because launch position has no impact on spacing between plane strikes.

Treat cants on surface perturbatively.

2 questions:
① How big do cants have to be to eliminate all parasitics?
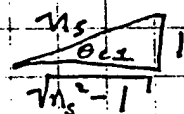② For a rectangular slab, how close to slab index does cladding index have to be to eliminate all parasitics.

This question will be easiest to answer for a zero loss parasitic

set z face incident angle equal to $\theta_{crit\,z} = \sin^{-1}\left(\frac{1}{n_s}\right)$ and the x face hit and y face hit also $= \theta_{crit\,z} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$
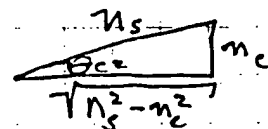
make this argument more rigorous!

Now work with direction cosines

$$\theta_{c_1} = \sin^{-1}\left(\frac{1}{n_s}\right)$$

$$\theta_{c_2} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$$

$$\cos\theta_{c_1} = \frac{\sqrt{n_s^2 - 1}}{n_s}$$

$$\cos\theta_{c_2} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos^2\theta_{c_1} + 2\cos^2\theta_{c_2} = 1$$

$$\frac{n_s^2 - 1}{n_s^2} + \frac{2(n_s^2 - n_c^2)}{n_s^2} = 1$$

$$n_s^2 - 1 + 2n_s^2 - 2n_c^2 = n_s^2$$

$$2(n_s^2 - n_c^2) = 1$$

$$n_s^2 - n_c^2 = \frac{1}{2}$$

when can this no longer be solved

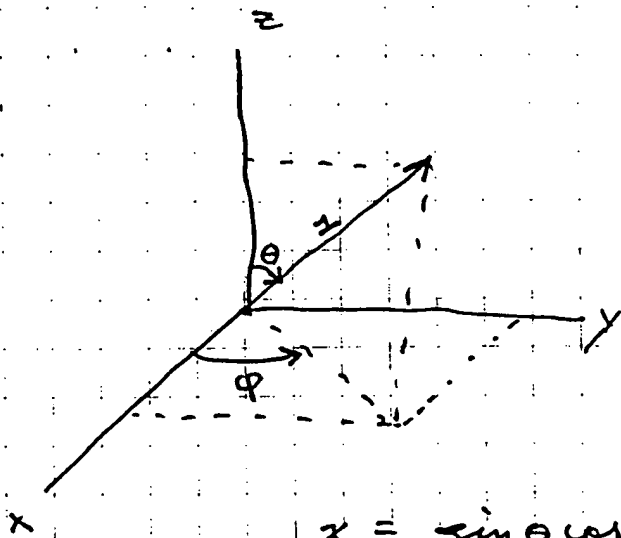$$n_c = \sqrt{n_s^2 - \frac{1}{2}}$$

$$n_c = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

for $n_c > 1.677$ no zero loss parasitics exist!

↳ this agrees with
code: slab ASE 02. XCL
prediction

Question 2 will be easiest to answer numerically:
finding the angular width over which
a parasitic exists for given gain and
cladding indices.

$$x = \sin\theta\cos\varphi = \cos\theta_x$$
$$y = \sin\theta\sin\varphi = \cos\theta_y$$
$$z = \cos\theta = \cos\theta_z$$

$$\begin{aligned}\theta_x &< \theta_{x\,crit}\\ \theta_y &< \theta_{y\,crit}\\ \theta_z &< \theta_{z\,crit}\end{aligned} \Bigg\} \quad \text{to avoid } 0\text{-loss parasitics}$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\begin{aligned}\cos\theta_x &> \cos\theta_{x\,crit}\\ \cos\theta_y &> \cos\theta_{y\,crit}\\ \cos\theta_z &> \cos\theta_{z\,crit}\end{aligned} \Bigg\rangle \quad \text{to avoid } 0\text{-loss parasitics}$$

$$\sin\theta_{x\,crit} = \frac{n_c}{n_s}$$
$$\sin\theta_{y\,crit} = \frac{n_c}{n_s}$$
$$\sin\theta_{z\,crit} = \frac{1}{n_s}$$

$$\cos\theta_x > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$
$$\cos\theta_y > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$
$$\cos\theta_z > \frac{\sqrt{n_c^2 - 1}}{n_s}$$

to avoid 0-loss parasi

$$1 = \cos\theta_x{}^2 + \cos\theta_y{}^2 + \cos\theta_z{}^2 \quad \text{and this must be greater than}$$

$$1 > \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

to avoid 0-loss parasiti

$$1 > \frac{3n_s^2 - 2n_c^2 - 1}{n_s^2}$$
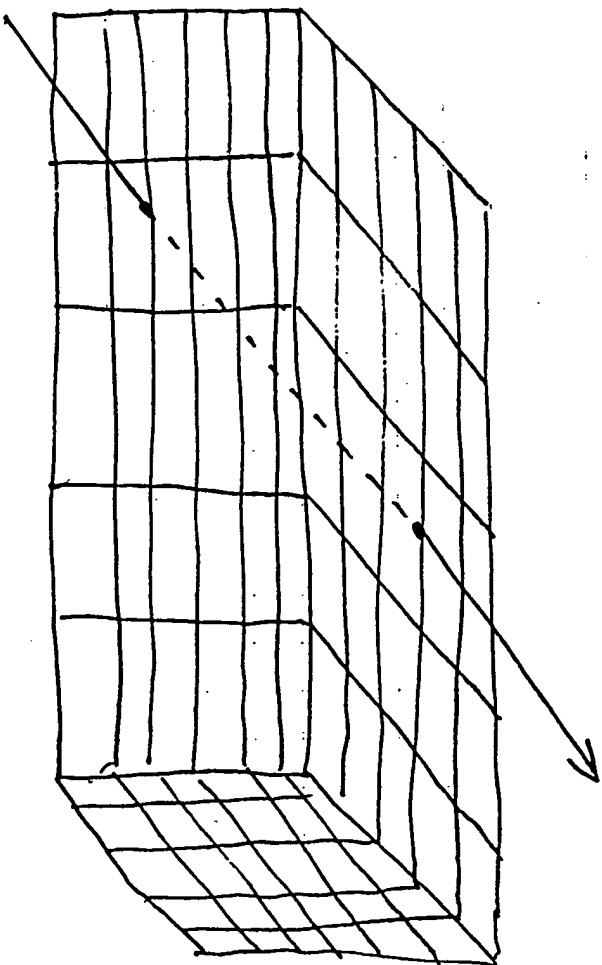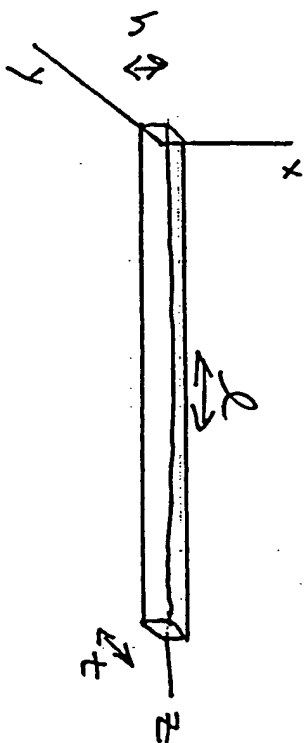
$$n_s^2 > 3n_s^2 - 2n_c^2 - 1$$

$$1 > 2\left(n_s^2 - n_c^2\right)$$

$$\frac{1}{2} > n_s^2 - n_c^2$$

$$n_c^2 > n_s^2 - \frac{1}{2}$$

$$n_c > \sqrt{n_s^2 - \frac{1}{2}} = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

n_s in slab index
n_c is coating index

fill space
using slab
and
scale images

o Define arbitrary ray direction using direction cosines
$(\cos\theta_x, \cos\theta_y, \cos\theta_z)$

• gain of ray is given in nepers/cm by

$$g = \frac{\ln(Ref_x)}{(h/\cos\theta_x)} \cdot \frac{\ln(Ref_y)}{(t/\cos\theta_y)} \cdot \frac{\ln(Ref_z)}{(\ell/\cos\theta_z)} \cdot \alpha$$

• where: Ref_i is the reflection coefficient for i-oriented planes

α is slab specific gain coating

zero-loss parasitics correspond to those rays may direction(s) that are confined by TIR at all three sets of planes

$$\cos\theta_x < \cos\theta_{x\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

TIR condition $\left\{\begin{array}{l} \cos\theta_y < \cos\theta_{y\text{-crit}} = \frac{n_s}{\sqrt{n_s^2 - n_c^2}} \\[2mm] \cos\theta_z < \cos\theta_{z\text{-crit}} = \frac{\sqrt{n_s^2 - 1}}{n_s} \end{array}\right.$

where:
n_s = slab ind.
n_c = coating index

○ Since $1 = \cos\theta_x^2 + \cos\theta_y^2 + \cos\theta_z^2$

$$1 < \frac{n_s^2 - n_c^2}{n_s^2} + \left(\frac{n_s^2 - n_c^2}{n_s^2}\right) + \frac{n_s^2 - 1}{n_s^2} \qquad \text{zero loss parasitics exist}$$

when

$$n_c < \sqrt{n_s^2 - \tfrac{1}{2}}$$

or

zero-loss parasitics can be completely suppressed by choosing a cladding with refractive index large enough

$$\boxed{n_c > \sqrt{n_s^2 - 1}}$$